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FINAL REPORT

SOLAR FLARE PHYSICS

March 21, 2000

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National Aeronautics and Space Administration

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Solar Flare Physics

In the extended portion of this grant (January 1-March 31, 2000), we have continued our previous efforts (January 1-December 31, 1999) in studies of Fourier imaging methods applied to hard X-ray flares. In particular, we have performed theoretical analysis of the "Forward-Fitting" method in collaboration with Dr. Markus Aschwanden, (Lockheed-Martin Palo Alto Research lab) in support of the HESSI (High Energy Solar Spectroscopic Imager) mission to enable rapid imaging of solar flares in hard X-rays.

The following pages outline our studies of the Forward Fitting method.

SECTION I
Speeding up Forward Fitting with Point Source Equivalents

Point-Source Equivalents

To speed up and de-pixelize the Forward Fitting program (M. Aschwanden, 1999), Gordon Hurford suggested we find the "Point-Source Equivalents" for elementary extended sources.

Rationale

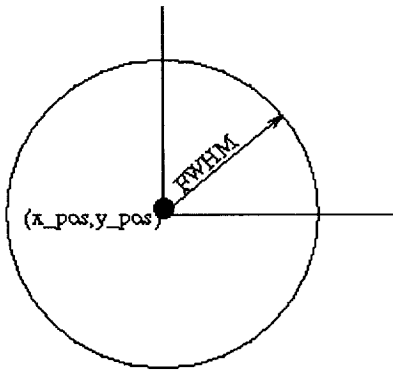
- Meshes well with existing simulation package, which accepts point sources as input
- Meshes well with visibility software
- Speeds up and generalizes existing Forward-Fitting code
- Permits elimination of pixelization in existing code

Examples of Elementary Extended Sources

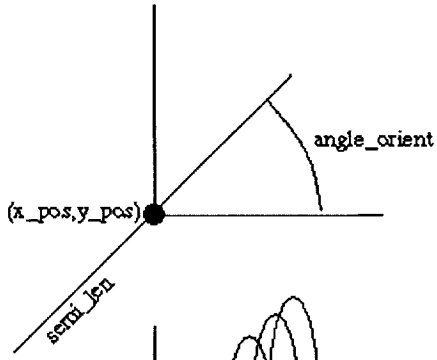
- Gaussians
  - Elliptical (flux, x_pos, y_pos, FWHM_1, FWHM_2, angle_orient)
  - Orthogonal (flux, x_pos, y_pos, FWHM_1, FWHM_2)
  - Circular (flux, x_pos, y_pos, FWHM)
- Segments
  - Simple (flux, x_pos, y_pos, semi_len, angle_orient)
  - Convolved (flux, x_pos, y_pos, semi_len, angle_orient, FWHM_1, FWHM_2)
- Arcs
  - Simple (flux, x_pos, y_pos, arc_radius, arc_halfangle, angle_orient)
  - Convolved
    (flux, x_pos, y_pos, arc_radius, arc_halfangle, angle_orient, FWHM_1, FWHM_2)

Figure 1 Source Objects

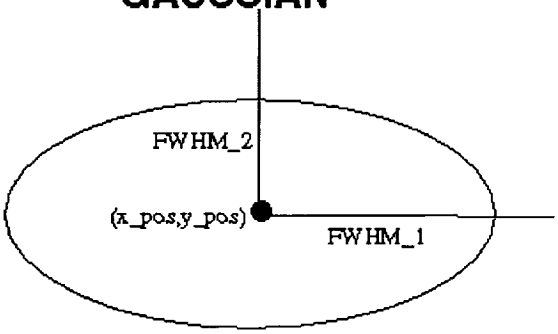
CIRCULAR GAUSSIAN



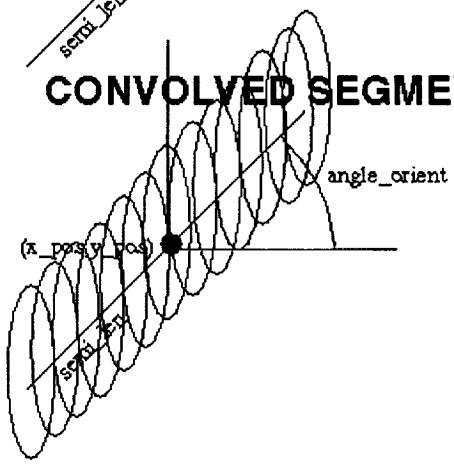
SIMPLE SEGMENT



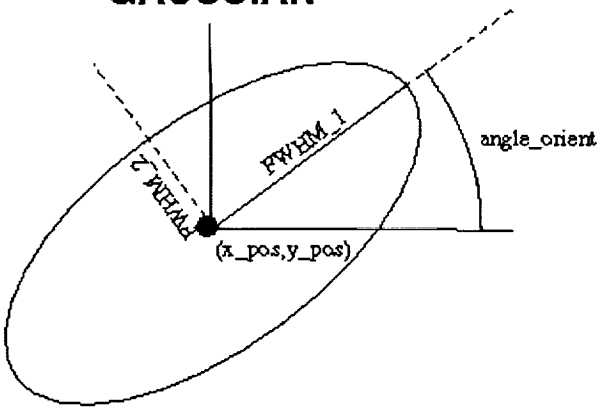
ORTHOGONAL GAUSSIAN



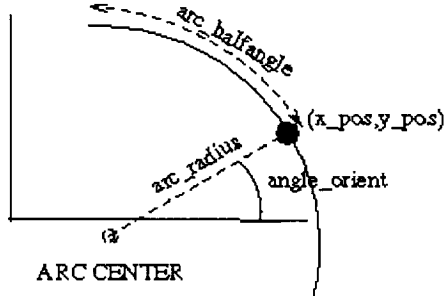
CONVOLVED SEGMENT



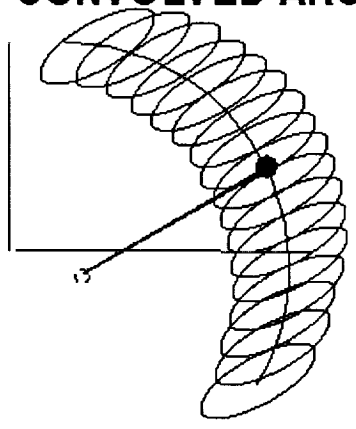
ELLIPTICAL GAUSSIAN



SIMPLE ARC



CONVOLVED ARC



The Mechanism

The mechanism behind this scheme is that for an arbitrary source, the visibility (*via* the Fourier transform) gives us:

- The amplitude and phase
- the location of a point source with the same phase,
- the flux of the equivalent point source with the same amplitude.

For all but the arcs, the Fourier transform is doable analytically, and the variables produced go straight into Forward Fitting of the count rate. This method works most simply for Gaussian sources and segments, for which

- the equivalent source location is the centroid of the source object itself;
- The flux of the equivalent source is independent of roll angle for:
  - circular Gaussian,
  - 360-degree arc,
  - any circularly-symmetric source;
- The flux of the equivalent source is roll dependent for:
  - orthogonal Gaussians
  - elliptical Gaussians
  - segments
  - arcs
  - almost anything else

For arcs and other such asymmetric shapes,

- the location of the equivalent point source is a function of roll angle;
- but this is not a complication, because so is the flux.
- The trick is to substitute a polygonal arc, and add up the contribution from segments;
- so the output of all the equivalent-source routines contains (at least) these vectors:
  - equivalent_flux
  - equivalent source_location
  - roll angle

Convolved Sources

- Any *Equivalent Point Source* (EPS) may be convolved with a Gaussian, producing another *Equivalent Point Source* .
- This is found very simply by a multiplication, angle bin by angle bin, of the equivalent flux for the input source and the Fourier amplitude of the convolving Gausssian.
- Any EPS can be convolved with a radially-symmetric source this way (*e.g.* an arc with a segment)
- An EPS can also be convolved with a non-radially-symmetric source (such as an arc), but
  - the phases must be included
  - why bother?

IDL programs for Equivalent Point Sources

Ed Schmahl
Last modified: Tue Mar 21 09:20:03 EST 2000

Index of /~schmahl/equiv_sources/

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Parent Directory	17-Mar-00 13:58	-	
equiv_gaussian.dvi	28-Feb-00 14:41	18K	
equiv_pt_srcs.html	21-Mar-00 12:24	1K	
fwd_fitting.eps	03-Mar-00 17:56	10K	
fwd_fitting.pdf	03-Mar-00 17:56	2K	
fwd_fitting.ps	03-Mar-00 17:56	11K	
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hsi_arc2.pro	03-Mar-00 17:56	4K	
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mechanism.html	21-Mar-00 12:27	2K	
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obsolete/	28-Feb-00 10:30	-	
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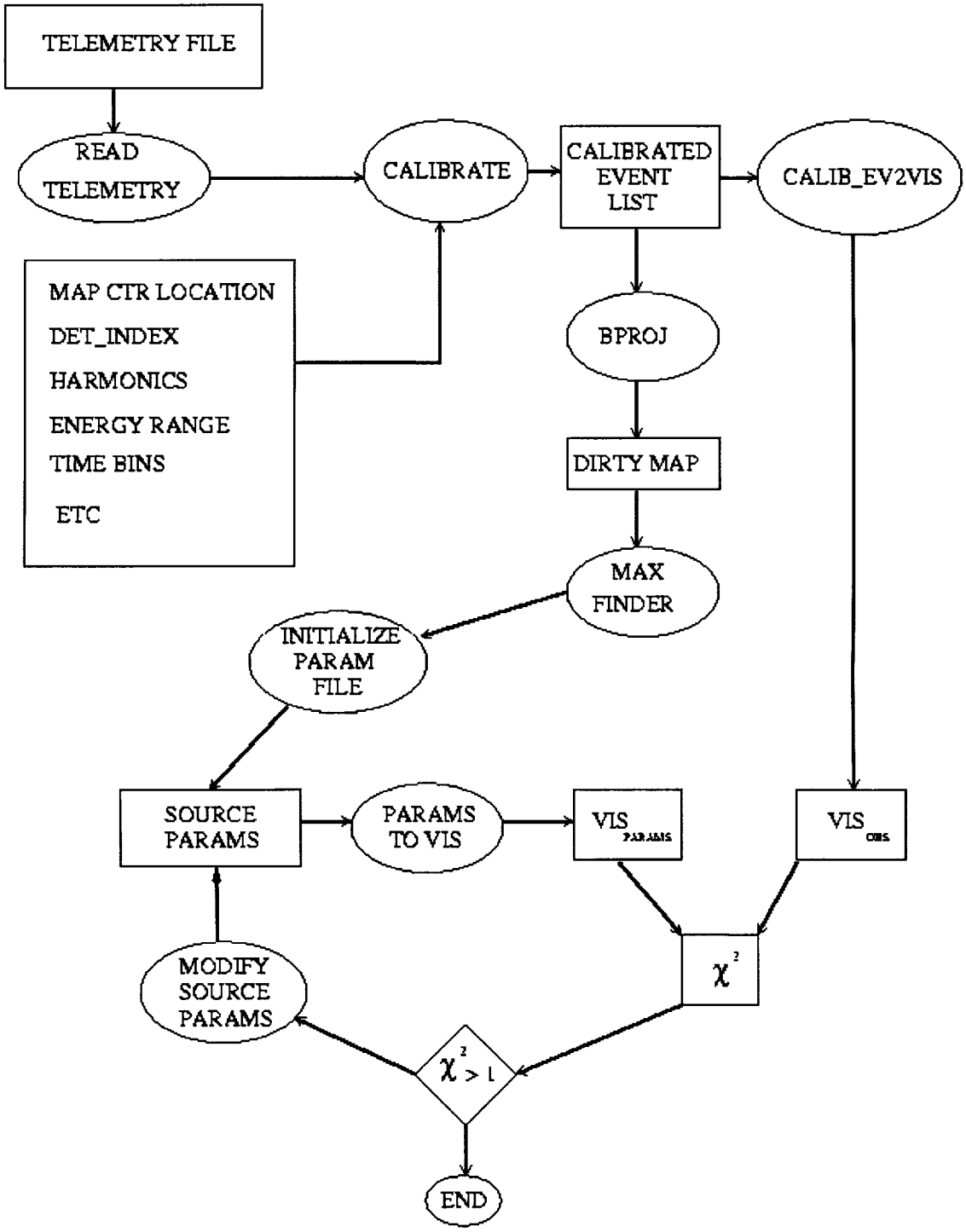
SECTION II

Schematic Diagram of the Forward Fitting Method


TELEMETRY FILE
READ TELEMETRY
CALIBRATE
CALIBRATED EVENT LIST
CALIB_EV2VIS
MAP CTR LOCATION
DET_INDEX
HARMONICS
ENERGY RANGE
TIME BINS
ETC
BPROJ
DIRTY MAP
MAX FINDER
INITIALIZE PARAM FILE
SOURCE PARAMS
PARAMS TO VIS
VIS PARAMS
VIS OBS
χ²
MODIFY SOURCE PARAMS
χ² > 1
END

SECTION III
Count Rates and Visibilities: How They Are Related

Count Rates and Visibilities

Given an incident flux of photons on a HESSI subcollimator, after the selection of a map center, the predicted count rate C is given by the following expression:

$$C = F_0\ T\ \tau\ \cdot \{1\ +\ c_1\ cos[\Phi - \Phi_1] + c_2\ cos[2(\Phi - \Phi_2)]\ +\ \cdots\} \quad (1)$$

where

$$\begin{aligned} F_0 &= \text{Incident photon flux on front grid} \\ T &= \text{Subcollimator transmission} \\ \tau &= \text{Livetime} \\ \Phi &= \text{Phase at map center} \\ c_n &= \text{Modulation Amplitude,} \quad n = 1,2,3,4,5 \\ \Phi_n &= \text{Modulation Phase,} \quad n = 1,2,3,4,5 \end{aligned}$$

In general, the phase at map center depends on the results of the aspect solution, the time binning, the subcollimator, and the position of the map center. The phase at map center Φ depends on the the projection of the vector between the subcollimator axis (x_{coll}, y_{coll}) and the map center (x_{map}, y_{map}):

$$\Phi = 2\pi\{[x_{map} - x_{coll}]cos\theta + [y_{map} - y_{coll}]sin\theta)/p\} \quad (2)$$

where

$$\begin{aligned} (x_{coll}, y_{coll}) &= \text{Subcollimator direction in inertial coordinates} \\ (x_{map}, y_{map}) &= \text{Map center in inertial coordinates} \\ \theta &= \text{Azimuth angle of slits} \\ p &= \text{Pitch of subcollimator grids} \end{aligned}$$

In the simplest case, where, the spin axis is at the origin and is stationary, the phase at map center is a function only of the azimuth ϕ, the pitch p, and the location of the map center (R_0, ϕ_0):

$$\Phi = 2\pi R_0 cos(\phi - \phi_0)/p \quad (3)$$

But in general, regardless of the position of the spin axis, the predicted count rate is:

$$C = F_0\ T\ \tau\ \cdot\{1\ +\ c_1\ cos\Phi\ +\ \cdots\} \tag{4}$$

The visibility of a point source at the map center is:

$$V = F_0\ e^{\mathrm{i}2\pi[x_{map}\ cos\theta + y_{map}\ sin\theta)]/p} \tag{5}$$

Note that the visibility does not depend on the subcollimator coordinates (x_{coll}, y_{coll}), but the count rate is subject to phase shifts caused by telescope motion. The phase shifts due to this motion are:

$$\Delta\Phi = 2\pi\{x_{coll}\ cos\theta + y_{coll}\ sin\theta)/p \tag{6}$$

And these are reflected in additional variations of the count rate profile. (The quantity $\Delta\Phi$ is computed from the aspect solution.) We can separate out this variation explicitly in the visibility:

$$V = F_0\ e^{\mathrm{i}[\Phi+\Delta\Phi]} \tag{7}$$

If we subtract out the mean of the count rate (equation 4), and divide out the $T\tau$ factor, keeping only the fundamental term,

$$(C - F_0\ T\ \tau)/(T\ \tau c_1) = F_0\ cos\Phi \tag{8}$$

So, for this particular case of a point source at map center the relation between the countrate profile and the visibilities are found by comparing equations 7 and 8:

$$Re\{Ve^{-\mathrm{i}\Delta\Phi}\} = \frac{C - F_0 T\tau}{c_1 T\tau} = \frac{C - < C >}{c_1 T\tau} \tag{9}$$

It is important to note that, by equation 9, the peak-to-peak variation of the visibility is several times larger than the peak-to-peak variation of C. This is because the visibility reflects the transform of the incident photons, while the count rate is the modulated profile of only the detected photons. In fact, when the livetime is unity ($T = 1$), and when the subcollimator pattern is perfectly triangular, ($c_1 = 8/\pi^2$ and $\tau = 1/4$), the peak-to-peak variation of $Re\{V\}$ is $\pi^2/2 = 4.9$ times that of the count profile.

For a more general case, where the source is an arbitrary superposition of N point sources of flux F_j and location (R_j, ϕ_j), the visibility is:

$$V = e^{\mathrm{i}\Delta\Phi} \sum_{j=1}^{N} F_j\ e^{\mathrm{i}\Phi_j} \tag{10}$$

where the Φ_j are the phases of the point sources at (R_j, ϕ_j).
Then, the corresponding predicted count rate is:

$$C = T\,\tau\,\cdot \sum_{j=1}^{N} F_j\,\{1\,+\,c_1\,cos[\Phi_j)]\,+\,\cdots\} \tag{11}$$

Comparing equations 10 and 11, we find that equation 9 remains valid, providing that F_0 is defined as the total flux of the N sources.

The generalization to higher harmonics is straightforward. The visibility for the n^{th} harmonic is given by:

$$Re\{V^{(n)}e^{-i\Delta\Phi}\} = \frac{C^{(n)}}{c_n T \tau} \tag{12}$$

where $C^{(n)}$ is the n^{th} term in the Fourier expansion of the count rate (equation 1). Each visibility profile derived from equation 12 then defines a portion of the entire Fourier transform obtained on a circle of radius n/p in the Fourier plane.